

Comparison of Simulation and Experimental Results of Travelling Wave JPA's in the Three Wave Mixing Regime

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Why Investigate Josephson Travelling Wave Parametric Amplifiers (JTWPA's)?

Josephson Parametric Amplifiers (JPA's) have long been popular for their **high gain** and **quantum limited noise**, although their narrow bandwidth has been a severe drawback. Josephson Travelling Wave Parametric Amplifiers (JTWPA's) are considered to be the **broadband successor** to these devices, covering the same applications as well as wide frequency range experiments such as qubit or MKID read-out. Recently, JTWPA's operating in the three-wave mixing (3WM) regime [1] have been sought after for the added advantages of easily separable pump and signal frequencies, **higher dynamic range** and **absence of self- (SPM) and cross-(XPM) phase modulation effects**. Although, effects of fabrication limitations [2] and shockwave generation [3] in the first generation of devices has hindered progress. This study aims to address the effects of fabrication limitations to inform the design choices of second generation and other future JTWPA devices.

How is Three-Wave Mixing (3WM) Achieved in JTWPA's?

- 3WM in parametric amplifiers obeys the relation:

$$\omega_{\text{pump}} = \omega_{\text{signal}} + \omega_{\text{idler}}$$

- Taylor expansion of current phase relation of a weak link has quadratic (3WM) and cubic (4WM) parts
- An rf-SQUID phase biased to a working point with infinite Josephson inductance, L_J allows for 3WM

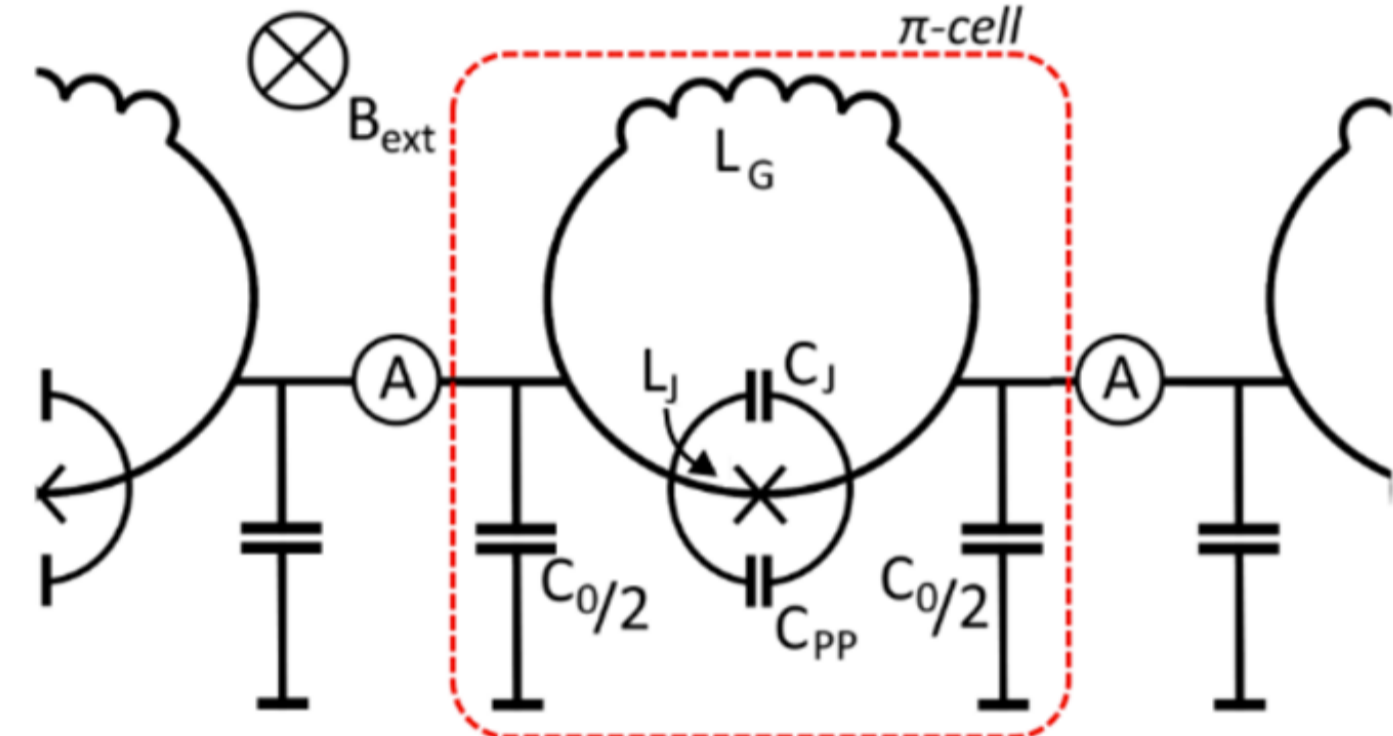


Fig. 1: Circuit diagram of an rf-SQUID element in the proposed 3WM JTWPA. Individual lumped elements can be used to accurately describe the device.

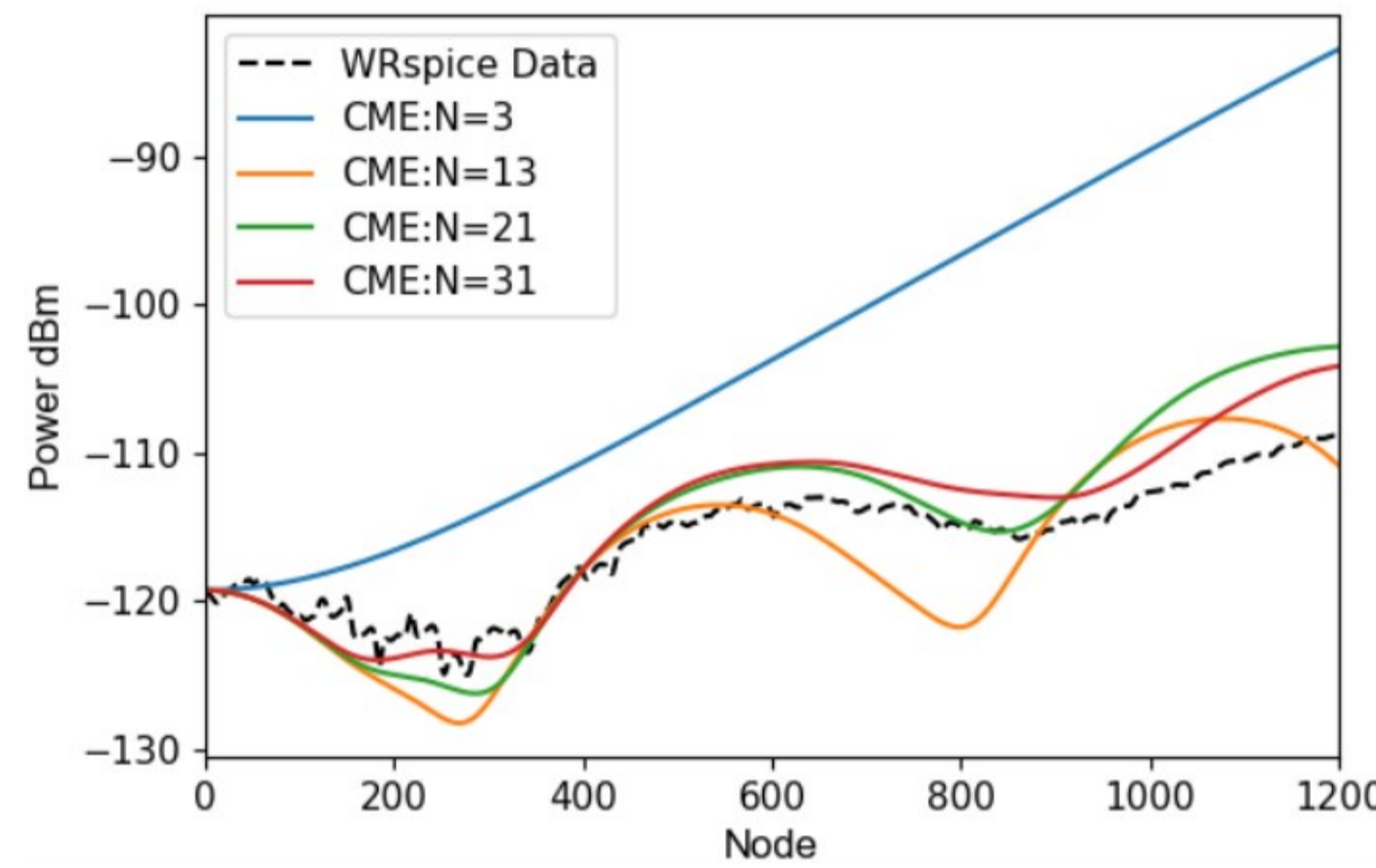


Fig. 2: Comparison of signal power profile along device from WRspice and CME solutions that consider N propagating waves in the device.

- When $\lambda \gg$ SQUID size, a , the circuit is approximately continuous
- Wave equations can be derived for the medium and Coupled Mode Equations describe wave mixing effects

- Fabricated samples are liable to parameter variation and defects, we study these in WRspice for the parameter set:

I_C	5uA
L_G	57pH
C_0	100fF
C_J	60fF

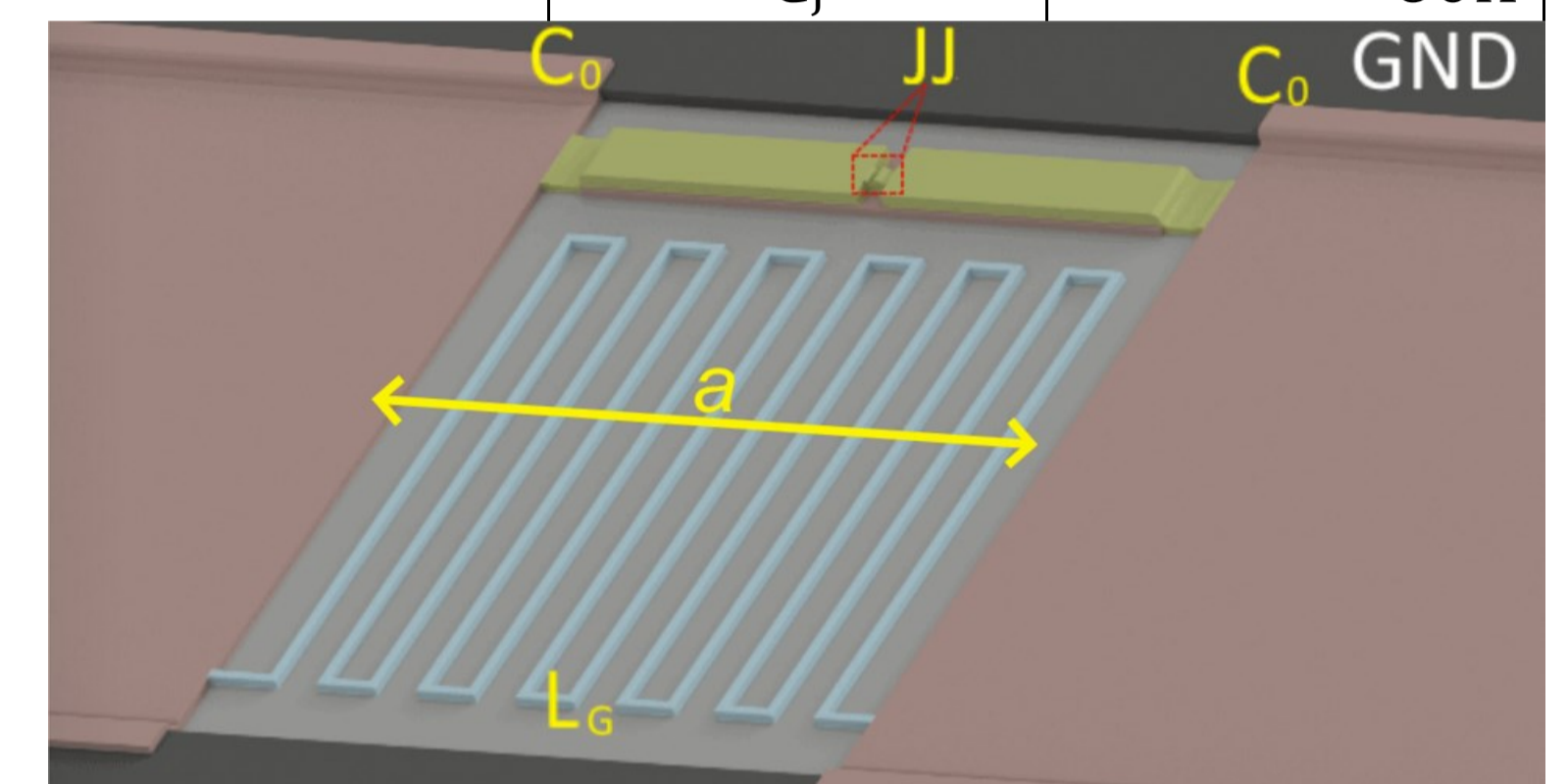


Fig. 3: Realistic schematic of an rf-SQUID element in a JTWPA device. Fabrication uncertainties are unavoidable.

How do Unavoidable Device Limitations Affect Performance?

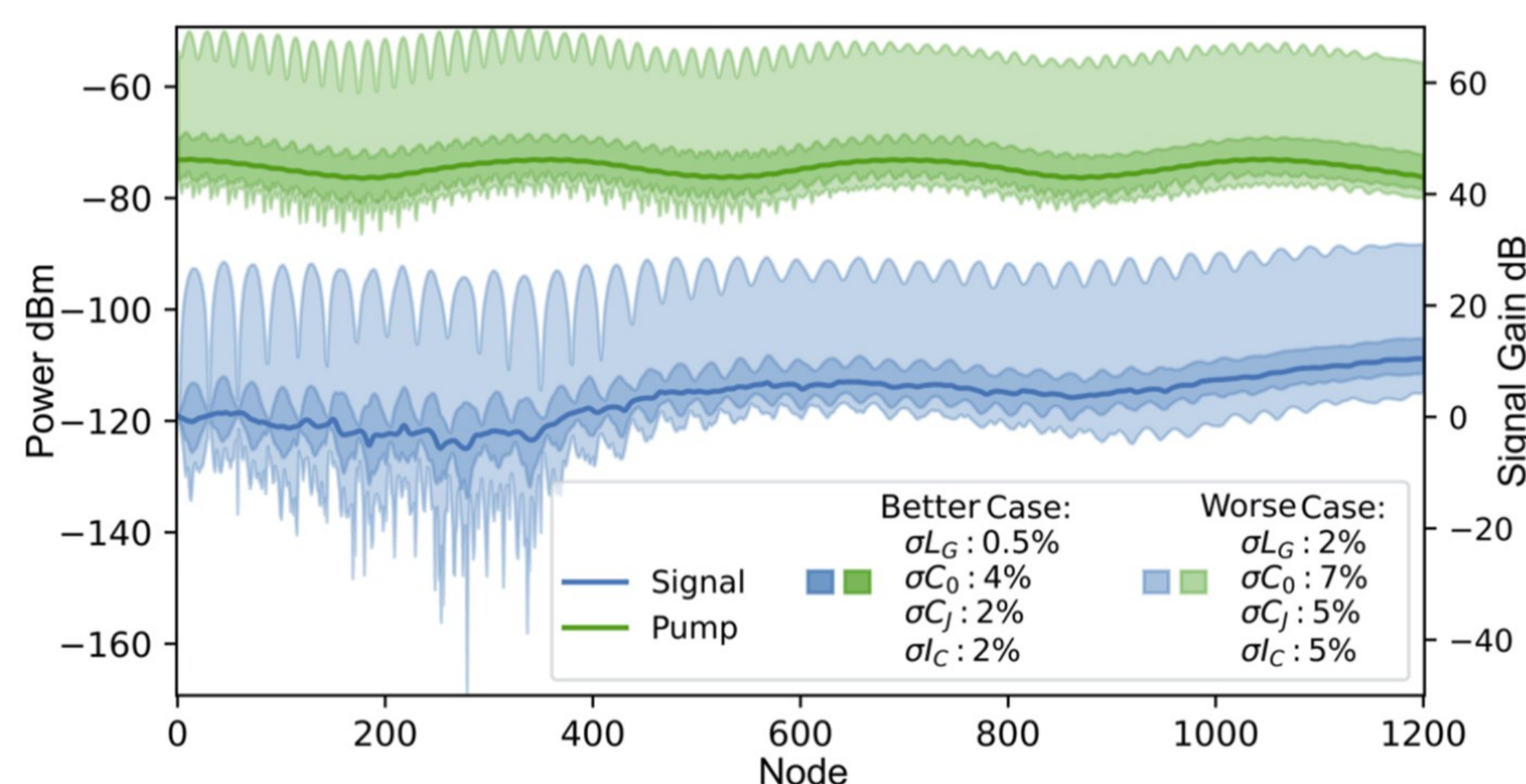


Fig. 4: Device profile showing signal (blue) and pump (green) tone powers at each cell in the array. Envelopes cover the area of possible values for particular amounts of variation, taken from a stochastic simulation method in WRspice.

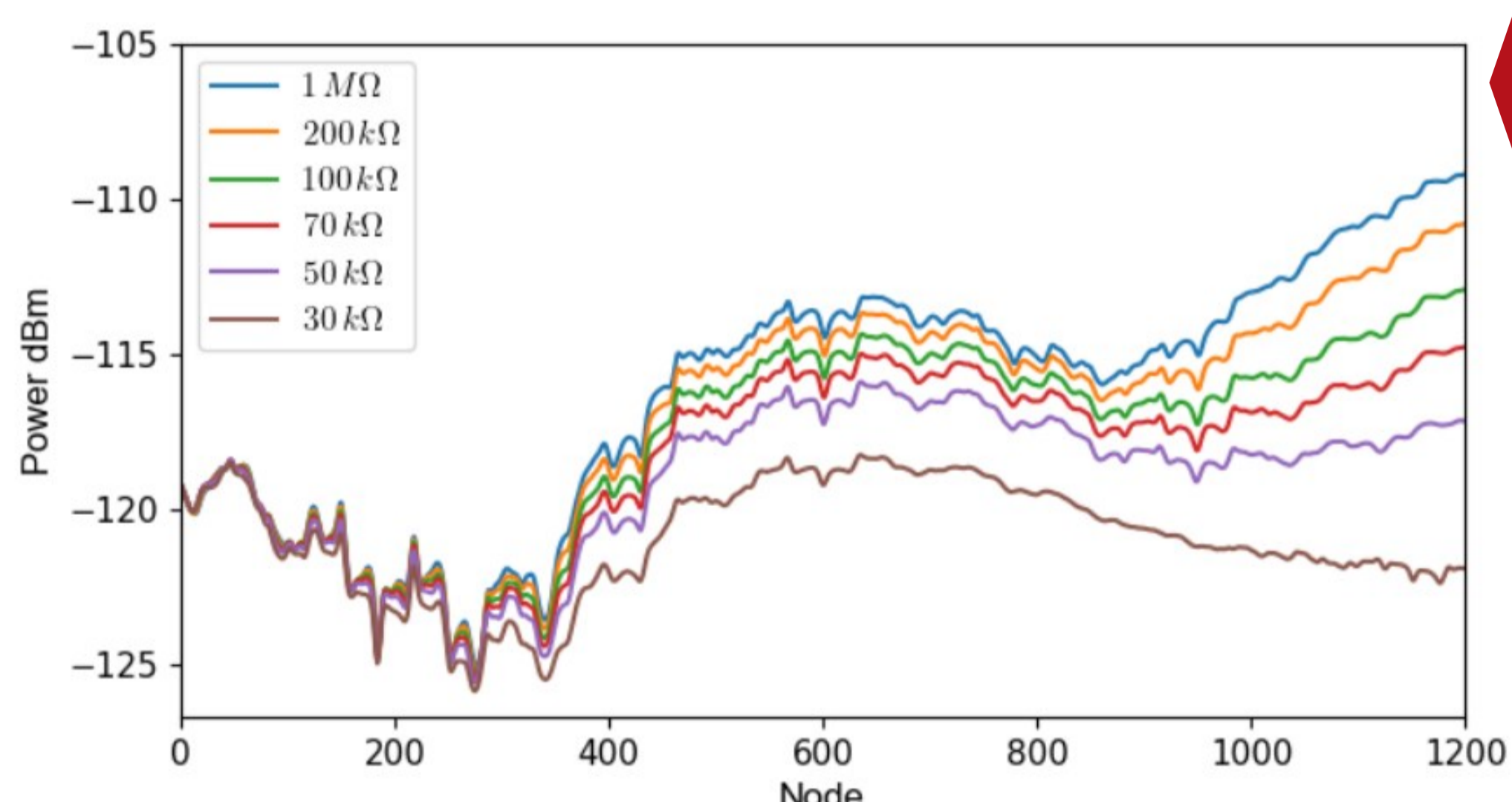


Fig. 6: Device profile showing the effects of changing the shunting resistance across the ground capacitance C_0 .

- Gaussian distributions of each parameter are simulated
- Envelopes of possible performance are created around ideal case profile

Parameter Variation

- Large impedance mismatches create internal resonances
- Leads to sharp spikes of high gain with narrow bandwidths in orange curve

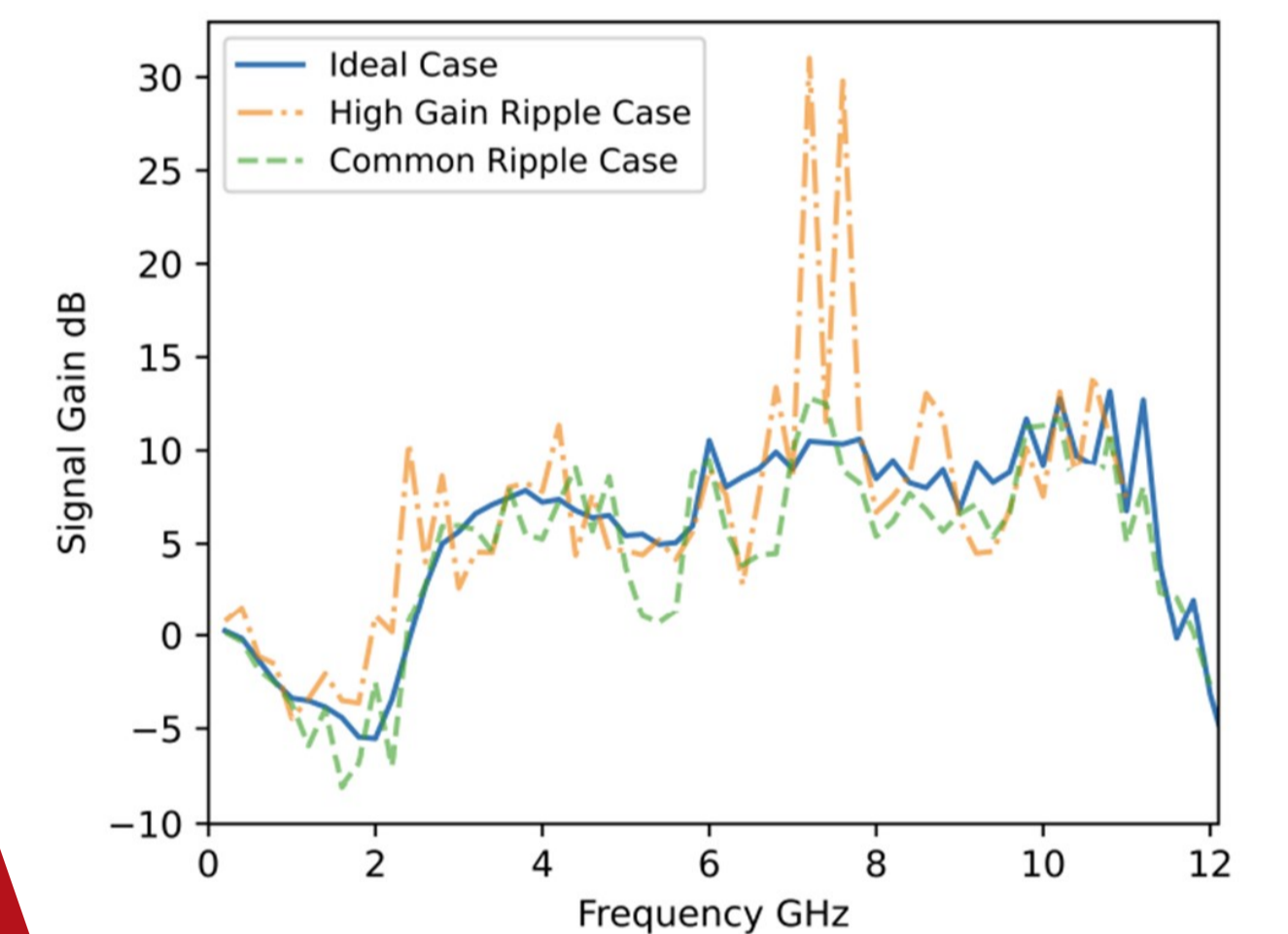


Fig. 5: Gain of signal tone found over a broad frequency range for an ideal case and two sets of varied parameters.

- Shunting the capacitance to ground with a resistance, R, leads to losses in the device and lower gain along the profile

Losses & Attenuation

- Across a frequency range the gain curve remains a similar shape with added losses
- Wave mixing occurs unperturbed but no gain is achieved

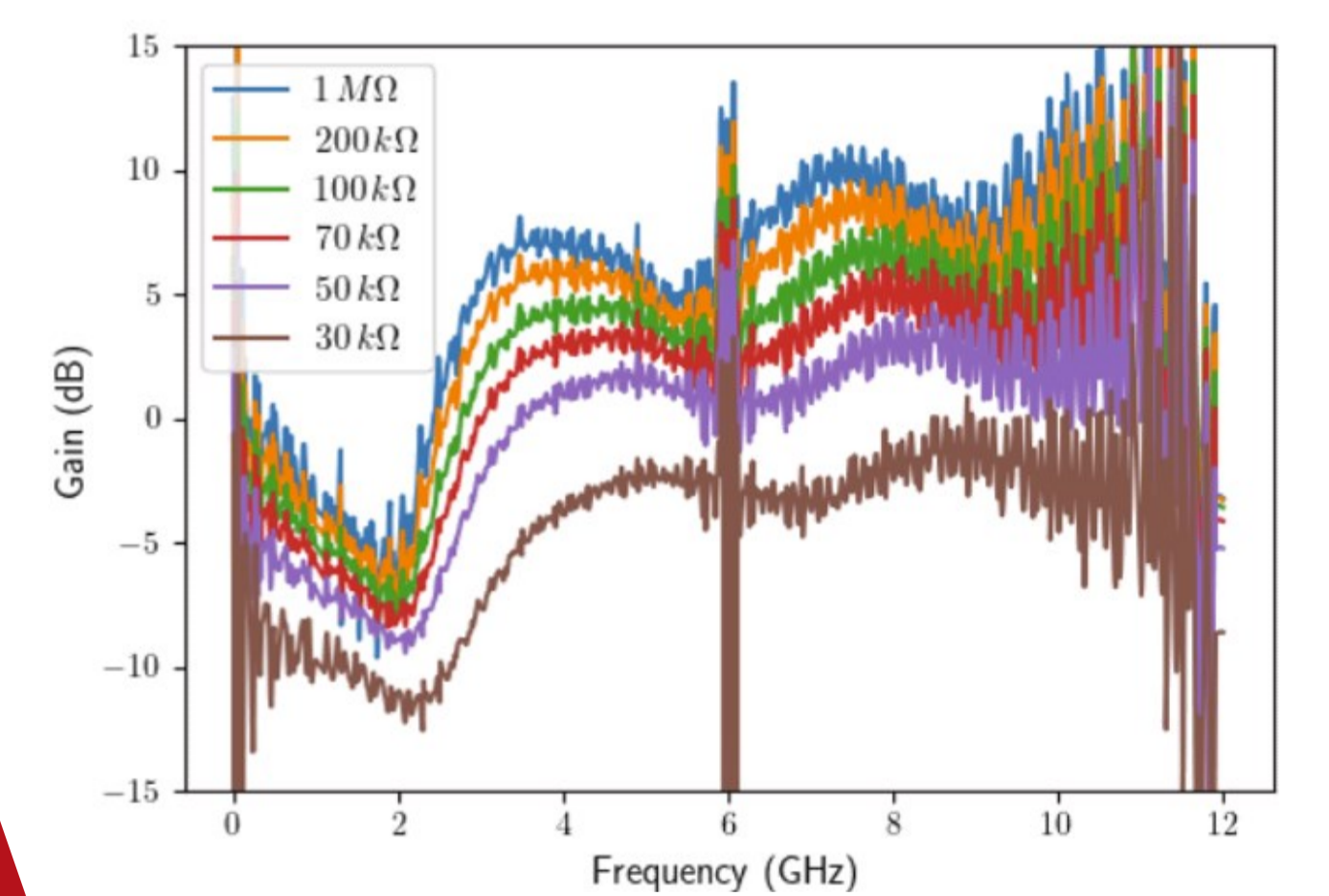


Fig. 7: Gain of signal tone over a broad frequency range showing the effects of decreased shunt resistance across ground capacitance, C_0 .

What Work Remains for Future Generations?

- Fabry-Perot ripples, shown in experimental results in Fig. 8, can be replicated in simulation to minimise effects in devices by better matching impedances
- Internal resonances, shown at ~ 6.5 GHz and ~ 6.85 GHz in Fig.8, should have their causes identified
- The effects of environmental and inter-SQUID flux coupling are not yet well modelled
- Dispersion engineering techniques can be modelled and optimized to enhance gain
- Transmission line geometries, e.g. Stripline or co-planar, can be analyzed and applied where appropriate

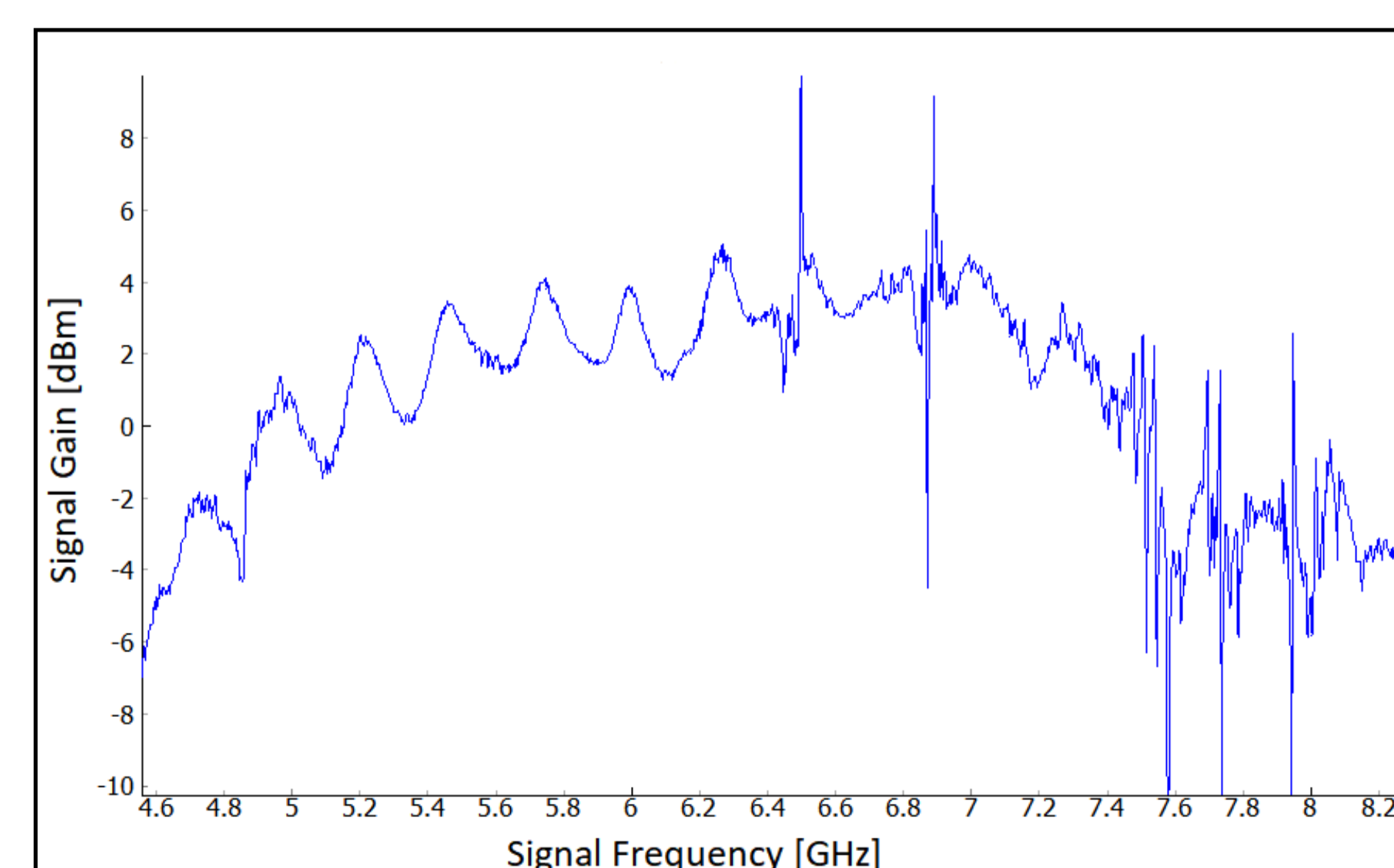


Fig. 8: Signal gain in device with Pump @ 15GHz, -70dBm and Signal @ -120dBm

For more information please see our paper:



[1] A.B. Zorin et al., Phys. Rev. Appl. 6, 034006 (2016).

[2] S.G. Ó Peatáin, Yu.A. Pashkin, et al. arXiv:2112.07766 (2021).

[3] T. Dixon, J.M. Williams, P.J. Meeson, et al. Phys. Rev. Appl. 14, 034058 (2020).

